

A) Select the correct answer

1) Only one of the following series diverges

$\sum \frac{(-1)^n}{n}$	$\sum \frac{4}{n^{\sqrt{8}}}$	$\sum \frac{4n}{n^6 + 1}$	$\sum \frac{4}{n^{1.4}}$	$\sum \frac{4}{\sqrt[5]{n}}$
Conv. By Alternate test	Conv. P-series $p=\sqrt{8} > 1$	Conv. Comparison test $b_n = \frac{4n}{n^6} = \frac{4}{n^5}$	Conv. P-series $p=1.4 > 1$	Diverge P-series $p=\frac{1}{5} \leq 1$

2)  $\sum_0^\infty \frac{9}{4^n} = 9 + \frac{9}{4} + \frac{9}{16} + \frac{9}{64} + \dots = \frac{a}{1-r} = \frac{9}{1-\frac{1}{4}} = 12$

Geometric series with  $a = 9, r = \frac{1}{4}$

5	10	12	6
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3) The sequence  $\left\{ \frac{4^n}{9^{n+1}} \right\}_{n=1}^\infty$

$\lim_{n \rightarrow \infty} \frac{4^n}{9^{n+1}} = \lim_{n \rightarrow \infty} \frac{4^n}{9^n \cdot 9} = \lim_{n \rightarrow \infty} \left(\frac{4}{9}\right)^n \cdot \frac{1}{9} = 0$  since  $\lim_{n \rightarrow \infty} a^n = \begin{cases} 0 & 0 < a < 1 \\ \infty & a > 1 \end{cases}$

- a) Diverge      b) converge to 5      c) converge to 0      d) converge to 1

4) The sequence  $\left\{ \frac{7^n}{3^{n+1}} \right\}_{n=1}^\infty$

$\lim_{n \rightarrow \infty} \frac{7^n}{3^{n+1}} = \lim_{n \rightarrow \infty} \frac{7^n}{3^n \cdot 3} = \lim_{n \rightarrow \infty} \left(\frac{7}{3}\right)^n \cdot \frac{1}{3} = \infty$

- a) Converge to 0      b) converge to 2      c) Diverge      d) converge to 3

B) Test the convergence of the following

1)  $\sum_0^\infty \frac{5^n n^2}{(n)!}$

$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \frac{5^{n+1}(n+1)^2}{(n+1)!} \cdot \frac{(n)!}{5^n n^2} =$

$\lim_{n \rightarrow \infty} \frac{5^n \cdot 5(n+1)^2}{(n+1)(n)!} \cdot \frac{(n)!}{5^n n^2} = \lim_{n \rightarrow \infty} \frac{5(n+1)}{n^2} = \lim_{n \rightarrow \infty} \frac{5n}{n^2} = \lim_{n \rightarrow \infty} \frac{5}{n} = 0 < 1$

Abs. Converge by Ratio test

$$2) \sum_1^{\infty} \frac{(2n+3)^n}{n^n}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{(2n+3)^n}{n^n}} = \lim_{n \rightarrow \infty} \frac{2n+3}{n} = 2 > 1$$

Diverge by root test

$$3) \sum_1^{\infty} \frac{(4n+2)^n}{n^{3n}}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{(4n+2)^n}{n^{3n}}} = \lim_{n \rightarrow \infty} \frac{4n+2}{n^3} = 0 < 1$$

Abs. Converge by Root test